Sample Question Paper - 11
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. In an AP, the first term is -4, the last term is 29 and the sum of all its terms is 150. Find its common difference. **[2]**

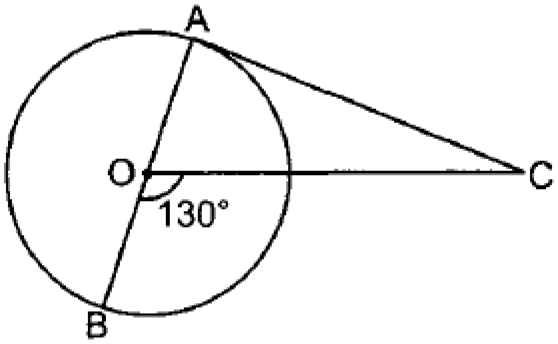
OR

If second term of an AP is (x - y) and 5^{th} term is (x + y), then find its first term.

2. Find the values of k for which the following equation has equal roots: **[2]**

$(k - 12)x^2 + 2(k - 12)x + 2 = 0$

3. In figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, then find $\angle ACO$. **[2]**



4. A well with inner radius 4 m is dug 14 m deep. Earth taken out of it has been spread evenly all around a width of 3 m it to form an embankment. Find the height of the embankment. **[2]**

5. Find the mean of the distribution: **[2]**

Class	1 – 3	3 – 5	5 – 7	7 – 9
Frequency	9	22	27	17

6. Find the roots of the quadratic equation by using the quadratic formula: **[2]**

$x^2 - 3\sqrt{5}x + 10 = 0$

OR

Find the value of k for which the roots of $9x^2 + 8kx + 16 = 0$ are real and equal.

Section B

7. Find the mean of the following frequency distribution: **[3]**

Classes	25-29	30-34	35-39	40-44	45-49	50-54	55-59
Frequency	14	22	16	6	5	3	4

8. Draw a line segment AB of length 5.4 cm. Divide it into six equal parts. Write the steps of construction. **[3]**

9. Calculate the mode of the following distribution: **[3]**

Class:	10-15	15-20	20-25	25-30	30-35
Frequency:	4	7	20	8	1

10. If a tower 30m high, casts a shadow $10\sqrt{3}m$ long on the ground, then what is the angle of elevation of the sun? **[3]**

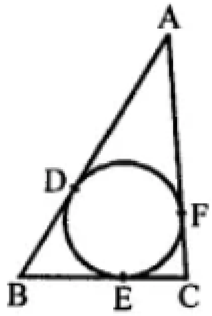
OR

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30^0 and 45^0. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between two ships.

Section C

11. A tent is of the shape of a right circular cylinder upto a height of 3 metres and then becomes a right circular cone with a maximum height of 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of ₹2 per square metre, if the radius of the base is 14 metres. **[4]**

12. In the given figure, a circle inscribed in a triangle ABC touches the sides AB, BC and CA at points D, E and F respectively. If AB = 14 cm, BC = 8 cm and CA = 12 cm. Find the lengths AD, BE and CF. **[4]**

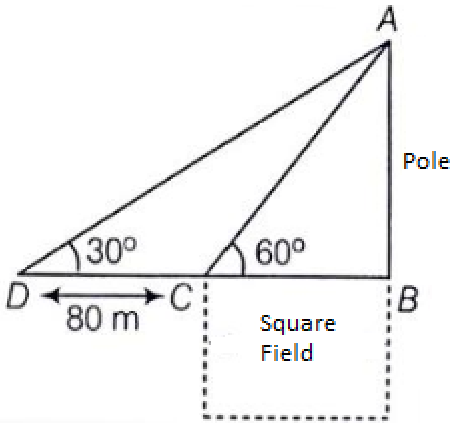


OR

Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

13. Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite **[4]**

to this corner is 60^{o}. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30^{o}.



i. Find the length of his square field so that he can buy material to do the fencing work accordingly.

ii. Find the height of the pole too so that he can arrange a ladder accordingly to put an effigy on the pole.

14. Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw material. **[4]**

They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find:

i. the production in the 1st year **(2)**

ii. the production in the 10th year **(1)**

iii. the total production in first 7 years **(1)**

Solution

MATHEMATICS STANDARD 041

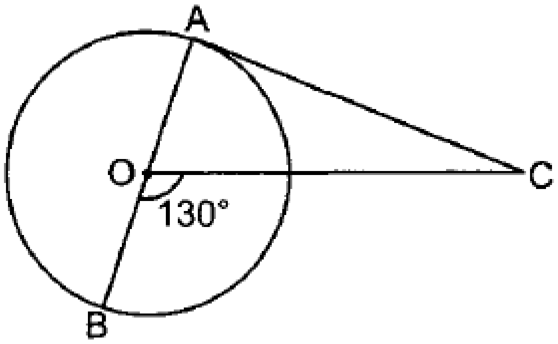
Class 10 - Mathematics

Section A

1. Let the given Arithmetic progression contains n terms.
   First term, a = -4
   Last term, l=29
   S_n = 150
   $\Rightarrow \frac{n}{2}$[a+l]=150
   $\Rightarrow \frac{n}{2}$[-4+29]=150
   $\Rightarrow$ n×25=300
   $\Rightarrow$ n = 12
   Thus, the given Arithmetic progression contains 12 terms.
   Let d be the common difference of the given Arithmetic progression .
   Then,
   T_{12} = 29
   But,T_{12}=a+(12-1)d
   $\Rightarrow$ a+11d=29
   $\Rightarrow$ -4+11d=29
   $\Rightarrow$ 11d=33
   $\Rightarrow$ d = 3
   Therefore common difference d is equal to 3

   OR

   $\because$ a + d = x - y ..(i)
   a + 4d = x + y ..(ii)
   Substracting (i) from (ii), we get
   $\Rightarrow$ -3d = -2y $\Rightarrow$ d = $\frac{2}{3}y$
   $\Rightarrow$ a + d = x - y
   $\Rightarrow$ a + $\frac{2}{3}y$ = x - y $\Rightarrow$ a = x - $\frac{5}{3}y$
2. We have, (k- 12)x^2 + 2(k - 12)x + 2 = 0.
   a = k - 12,b = 2(k - 12) and c = 2.
   The given equation will have equal roots, if
   $\therefore$ D = b^2 - 4ac = 0
   $\Rightarrow$ 4(k - 12)^2 - 4 (k - 12)$\times$ 2 = 0
   $\Rightarrow$4(k- 12) [(k - 12) - 2] = 0
   $\Rightarrow$4(k - 12)(k - 14) = 0
   $\Rightarrow$4(k - 12) (k - 14) = 0 $\Rightarrow$ k - 12 = 0 or, k - 14 = 0 $\Rightarrow$ k = 12 or, k = 14
3. According to the question, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A



   $\angle AOC = 180^\circ - 130^\circ = 50^\circ$
   $\angle OAC = 90^\circ$ [$\because$OA $\perp$ AC]
   In $\triangle$OAC,

$\angle OAC + \angle AOC + \angle ACO = 180^\circ$
$90^\circ + 50^\circ + \angle ACO = 180^\circ$
$\Rightarrow \angle ACO = 40^\circ$

4. Radius of well = 4 m
Depth of the well = 14 m
Width of embankment = 3 m
Radius of outer surface of embankment = 3 + 4 = 7 m
Let height of embankment = h m
According to the question,
Volume of embankment = Volume of earth dug out
$\Rightarrow \quad \pi\left(7^2 - 4^2\right) \times h = \pi(4)^2 \times 14$
$\Rightarrow \quad (49 - 16) \times h = 16 \times 14$
$\Rightarrow \quad 33h = 16 \times 14$
$\Rightarrow \quad h = \frac{16 \times 14}{33} = 6.78m$
Therefore, Height of embankment = 6.78 m

5.

Class	**Class mark (x_i)**	**Frequency (f_i)**	f_ix_i
1 – 3	2	9	18
3 – 5	4	22	88
5 – 7	6	27	162
7 – 9	8	17	136
		$\sum f_i = 75$	$\sum f_ix_i = 404$

Mean $\overline{x} = \frac{\sum f_ix_i}{\sum f_i} = \frac{404}{75} = 5.3$

6. Given, $x^2 - 3\sqrt{5}x + 10 = 0$
By using quadratic formula
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$\therefore \text{x} = \frac{-(-3\sqrt{5}) \pm \sqrt{(-3\sqrt{5})^2 - 4(1)(10)}}{2(1)}$
$= \frac{3\sqrt{5} \pm \sqrt{5}}{2} = 2\sqrt{5}, \sqrt{5}$

OR

The given equation is $9x^2 + 8kx + 16 = 0$
Comparing it with, $ax^2 + bc + c = 0$, we have
$a = 9,\ b = 8k\ and\ c = 16$
Now, for equal roots, we have
Discriminant, D = 0
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow (8k)^2 - 4(9)(16) = 0$
$\Rightarrow 64k^2 - 576 = 0$
$\Rightarrow 64k^2 = 576$
$\Rightarrow$ k^2 = $\frac{576}{64}$
$\Rightarrow$ k^2 = 9
$\Rightarrow$ k = $\pm$ 3
$\Rightarrow$ k = 3 or k = -3
Hence, the required value of k is 3 ,-3.

Section B

7.

Class interval	**Mid value x_i**	**d_i= x_i – 42**	$u_i = \frac{(x_i - 42)}{5}$	**Frequency f_i**	f_iu_i
25 – 29	27	-15	-3	14	-42
30 – 34	32	-10	-2	22	-44

35 – 39	37	-5	-1	16	-16
40 – 44	42	0	0	6	0
45 – 49	47	5	1	5	5
50 – 54	52	10	2	3	6
55 – 59	57	15	3	4	12
				$\sum f_i$ = 70	$\sum f_i u_i$ = -79

Let the assumed mean (A) = 42

h = 5

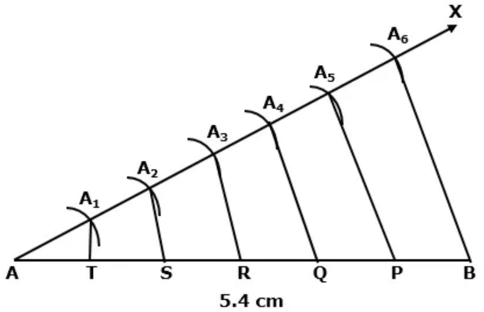
Mean = A + h$\frac{\sum f_i u_i}{\sum f_i}$

= 42 + 5$\left(\frac{-79}{70}\right)$

= 42 - $\frac{79}{14}$

= 36.357

8.



Steps of construction:

1. Draw a line segment AB = 5.4 cm.
2. Draw a ray AX making an acute $\angle$BAX with AB
3. Along AX mark 6 points A_1, A_2, A_3, A_4, A_5, A_6, such that

   $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6$
4. Join A_6B.
5. Through point A_5, draw a line parallel to A_6B by making an angle equal to $\angle AA_6B$ at A_5.

   Suppose this line meets AB at a point P.
6. Similarly, through points, A_4, A_3, A_2, A_1, draw lines parallel to A_6B

   Suppose these line meet AB at points Q, R, S, T respectively.
   Thus, line segment AB is divided into 6 equal parts such that
   AT = TS = SR = RQ = QP = PB

9.

Class Interval	f
10 - 15	4
15 - 20	7
20 - 25	20
25 - 30	8
30 - 35	1

l = 20 f_1 = 20 f_o = 7 f_2 = 8

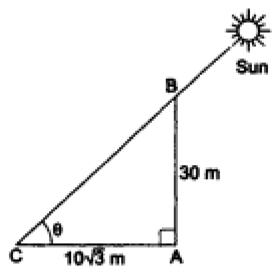
Mode = l + $\left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right]$ h

= 20 + $\left[\frac{20-7}{2\times20-7-8}\right]$ 5

= 20 + $\left[\frac{13}{40-7-8}\right] \times 5$

$= 20 + \left[\frac{13}{25}\right] \times 5$
$= \frac{100+13}{5} = \frac{113}{5} = 22.6$

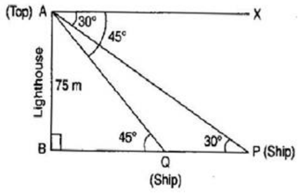
10. Let AB be the pole and let AC be its shadow.



Let the angle of elevation of the sun be θ°.
Then, $\angle ACB = \theta, \angle CAB = 90^{\circ}$.
$AB = 30m$ and AC = $10\sqrt{3}m$.
From right ΔCAB, we have
$\tan\theta = \frac{AB}{AC} = \frac{30}{10\sqrt{3}} = \sqrt{3}$
$\Rightarrow \theta = 60^{\circ}$

OR

In right triangle ABQ,

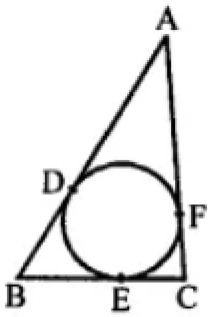


$\tan 45^{\circ} = \frac{AB}{BQ}$
$\Rightarrow 1 = \frac{75}{BQ}$
$\Rightarrow$BQ = 75 m .... (i)
In right triangle ABP,
$\tan 30^{\circ} = \frac{AB}{BP}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BQ+QP}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{75+QP}$ [From eq. (i)]
$\Rightarrow$ 75 + QP = $75\sqrt{3}$
QP = $75(\sqrt{3}-1)$ m
Hence the distance between the two ships is $75(\sqrt{3}-1)$ m.

Section C

11. Height of the cylinder = 3 m.
Total height of the tent above the ground = 13.5 m
height of the cone = (13.5 - 3)m = 10.5 m
Radius of the cylinder = radius of cone = 14 m
Curved surface area of the cylinder = $2\pi rh\text{m}^2 = \left(2 \times \frac{22}{7} \times 14 \times 3\right)\text{m}^2 = 264\text{m}^2$
$\therefore \quad l = \sqrt{r^2 + h^2} = \sqrt{14^2 + (10.5)^2} = \sqrt{196 + 110.25} = \sqrt{306.25} = 17.5$
$\therefore$ Cured surface area of the cone = $\pi rl = \left(\frac{22}{7} \times 14 \times 17.5\right)\text{m}^2 = 770\text{m}^2$
Let S be the total area which is to be painted.Then,
S = Curved surface area of the cylinder + Curved surface area of the cone
$\Rightarrow$ S = (264 + 770) m^2 = 1034 m^2
Hence, Cost of painting = S $\times$ Rate = Rs(1034×2)= ₹2068

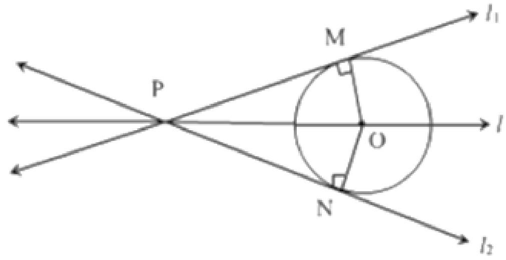
12. In the given figure, a circle is inscribed in a ΔABC touches the sides AB, BC and CA at D, E and F respectively.
AB = 14 cm, BC = 8 cm and CA = 12 cm.



To find : The length of AD, BE and CF.
Let AD = x, BE = y and CF = z
AD and AF are the tangents to the circle from A.
AD = AF = x
Similarly,
BE and BD are tangents
BD = BE = y
and CF and CE are the tangents
CE = CF = z
Now, AB + BC + CA = 14 + 8 + 12 = 34
=> (x + y) + (y + z) + (z + x) = 34
=> 2 (x + y + z) = 34
=> x + y + z = 17 ...(i)
But x + y = 14 cm ...(ii)
y + z = 8 cm ...(iii)
z + x = 12 cm ...(iv)
Subtracting (iii), (iv) and (ii) from (i) term by term
x = 17 – 8 = 9 cm
y = 17 – 12 = 5 cm
z = 17 – 14 = 3 cm
Hence, AD = 9 cm, BE = 5 cm and CF = 3 cm.

OR

Let l_1 and l_2 be two intersecting lines.



Suppose a circle with centre O touches the lines l_1 and l_2 at M and N respectively.

Therefore,OM = ON

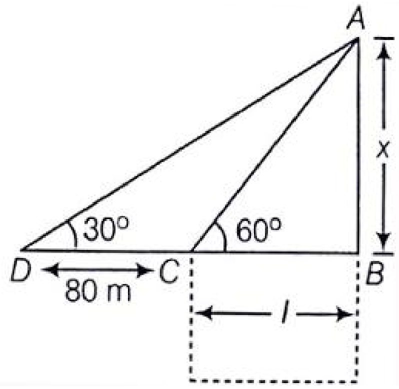
Therefore, O is equidistant from l_1 and l_2.

Consider ΔOPM and ΔOPN,

$\angle OMP = \angle ONP$ ....(radius is perpendicular to the tangent)
OP = OP ...(Common side)
OM = ON ...(radii of the same circle)
$\Rightarrow \Delta OPM \cong \Delta OPN$ ...(RHS congruence criterion)
$\Rightarrow \angle MPO = \angle NPO$ ...(CPCT)
$\Rightarrow$ l bisects $\angle MPN$.
$\Rightarrow$ O lies on the bisector of the angles between l_1 and l_2, that is, O lies on l.

Therefore, the centre of the circle touching two intersecting lines lies on the angles bisector of the two lines.

13. The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

i. In △ABC,

$\tan 60^o = \frac{x}{l}$

$\sqrt{3} = \frac{x}{l}$

x = $\sqrt{3}$ l ......(1)

Now, in △ABD,

tan30o = $\frac{x}{80+l}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}l}{80+l}$ (From eq(1))

80 + l = 3l

2l = 80

l = 40

Thus, length of the field is 40 metres.

ii. Now, l = 40 metres

Thus from eq(1), we get,

x = $\sqrt{3}$ l = 40$\sqrt{3}$ = 69.28

Thus, height of the pole is 69.28 metres.

14. Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We have,a_3 = 600 and a_7 = 700 ⇒ a + 2d = 600 and a + 6d = 700. Solving these equations, we get;a = 550 and d = 25.

1. We have, a = 550

∴ Production in the first year is of 550 TV sets.

2. The production in the 10th term is given by a_{10}.

Therefore, production in the 10th year = a_{10} = a + 9d = 550 + 9 × 25 = 775. So, production in 10th year is of 775 TV sets.

3. Total production in 7 years

= Sum of 7 terms of the A.P. with first term a (= 550) and common difference d (= 25).

$= \frac{7}{2}\{2 \times 550 + (7-1) \times 25\}$

$= \frac{7}{2}(1100 + 150) = 4375.$